AN APPROACH TO THE ISOLATION OF SENSOR AND ACTUATOR FAULTS BASED ON SUBSPACE IDENTIFICATION

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Abstract

This paper addresses the problem of sensor and actuator fault isolation in MIMO discrete-time linear time-invariant systems. The considered class of faults are represented by scaling factors for both the control signals and the measured (output) signals, and constant offset terms. The starting point is the identification of a state-space model, that represents the deterministic part of the faulty system up to a similarity transformation, from corrupted by noise input output measurements. Based on the obtained model we derive a procedure for the isolation of the faults in the system. It first solves a suitably defined non-linear optimization problem over the scaling factors. Necessary condition for the existence of unique solution to the optimization problem is given, and then the unique global optimizer is derived in analytical form. Given the optimal scaling factors we next find the similarity transformation that transforms the identified model to the initial state basis, which is a necessary step for the determination of the offset factors. The approach is verified in a simulation study with a linearized model of the lateral directional dynamics of an F-14 aircraft, in which simultaneous faults in all sensors and actuators were simulated.

keywords: fault isolation, subspace identification, non-linear least squares

1 INTRODUCTION

System faults are unanticipated events which abruptly change the behaviour of the control system, degrading the overall system performance, and often leading to major failures. Thus, simple faults, causing malfunctioning of sensors, actuators or components of the system should be swiftly diagnosed, and appropriate remedial actions [5, 16, 17, 10, 7] should be undertaken, in order to prevent their development into serious system failures. Such failures in systems that operate in space [6, 11, 23] could lead to huge financial and even human losses. It is therefore of crucial importance that such complex systems should have increased availability, survivability and maintainability, which could be achieved through
d fault detection and isolation (FDI), followed by appropriate controller reconfiguration.

In this paper we discuss the problem of isolation of simultaneous sensor and actuator faults given corrupted by noise input/output measurements from the faulty system. The faulty MIMO discrete time-invariant system is assumed to have resulted from sensor and actuator faults. We model these faults by scaling factors for the inputs and the outputs plus
offset terms in the state and in the measurement equations. The Subspace Model Idenification (SMI) method is used to identify a state-space model of the faulty system. Using the identified model, which is in a different basis from the nominal state-space model, we derive a procedure for isolation of the faults in the system. It first finds the magnitudes of the scaling factors, which is a non-linear optimization problem that is independent on the state basis. We derive necessary and sufficient conditions for the existence of a unique solution to the non-linear optimization problem, and then we find the unique solution in an analytical form. Given the estimated magnitudes we next find the similarity transformation that transforms the estimated state-space model to the initial state basis, which is needed to obtain the offset terms in the state and in the output equations. The method is verified in a simulation example with a linearized model of the lateral directional dynamics of an F-14 aircraft, in which simultaneous faults in all sensors and all actuators were simulated. The input/output measurements in this case study were corrupted by 16dB measurement noise, and 25dB process noise.

In general, there are two main tasks that should be performed by a fault-tolerant control system [15, 18, 28] after any occurrence of a fault in the system. The first one is the fault detection and isolation (FDI) task [10, 3, 25, 24, 14], and the one that follows is the controller reconfiguration (CR) task [16, 13, 26]. The main stream of research is focused on each of these two tasks. Recently a number of papers have started to address the interaction between FDI and CR

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[15, 8, 9, 13, 20, 26]. Even for linear systems the task of detection and isolation of a combination of (sensor, actuator, and component) faults remains a tangible problem both from an engineering and from system theoretical perspective. For an overview of the research field of FDI for linear systems we refer to [3, 10, 16, 28].

Here a majority of contributions focus on the detection of particular classes of faults. There is either sensor fault detection [19, 15], where sensor faults are modeled by an unknown vector $f_s$ added to the measurement equation, or actuator faults [8, 24, 15], modeled by adding a vector $f_a$ to the dynamics equation. The detection and isolation of a combination of sensor, actuator and component faults is much more difficult. One strategy is the multi-model approach where the database is constructed representing a finite number of (anticipated) faults. Examples of this approach are [13, 26, 27, 2, 11, 13, 4].

The approach in [4] is based on subspace identification. The detection corresponds to a statistical analysis of a generated residual that measures the difference between the nominal and the actual identified model that possibly represents the faulty system behavior. The isolation in this approach consists in measuring the difference between the actual behavior and the models in the database. The later only represents a finite number of specific faulty conditions.

In this paper we also apply subspace identification methods [21] to identify a model of the actual system behavior (as represented in a batch of input/output data). Knowing a model of the actual system behavior we then attempt to match a parametrized model describing the faults in the nominal system to this actual system model. Though in general this gives rise to a non-linear parameter optimization problem, we show in this paper how to derive an elegant and analytically tractable parameter estimation problem for the case of considering a combination of sensor and actuator faults. The approach is applicable to the isolation of any combination of sensor and actuator faults (which is an infinite set of faults).

The FDI scenario considered is that a batch of input/output measurements is, at regular time instances, transmitted for analyzing whether faults have occurred, and if so, what their magnitudes are. Such problems are extremely relevant in space applications (e.g. in robotic space manipulators [6]), where batches of input/output data are transmitted periodically from space to ground station for analysis.

The organization of the paper is as follows. In the next section we will start by defining the mathematical parametrization of the sensor and actuator faults given a state-space model representing the nominal system behavior. Subsequently, the FDI problem addressed in this paper is formulated. Section 3 presents isolability conditions for the isolation of simultaneous sensor and actuator faults given input/output data. Then the FDI problem is formulated as a constraint non-linear parameter estimation problem, of which a solution can be determined analytically. Section 4 presents the overall solution to the considered FDI problem based on an identified model by subspace identification to represent the actual system behavior. A simulation example, illustrating the theory in the paper is given in section 5. Finally, some concluding remarks are made in section 6.

2 PROBLEM FORMULATION

In this section we define the different types of faults, which can be attacked by the proposed approach.

Faulty Model Parametrization

In the fault-tolerant control literature different types of faults are differentiated. Abrupt faults are faults that result in abrupt changes of the system’s behaviour. These faults are further subdivided into total sensor/actuator faults (representing complete loss of instrumentation), partial sensor/actuator faults (representing partial loss of instrumentation), and component faults (representing abrupt change in a physical parameter (component) of the system). Incipient faults represent gradual changes in the components of the system. Sensor faults represent incorrect reading from the sensors, while actuator faults represent loss of control action. In this paper the faults are assumed to be constant in the data batch received. Incipient faults will then appear by observing a gradual change in the fault magnitudes analyzed from different data batches over time.

A large class of linear faults in the actuators of the system can be represented by scaling the control input $u(t)$ and adding an additional offset term, that cannot be manipulated (i.e. it cannot be used for steering of the system). Similarly, linear sensor faults can be modeled by scaling the measured output of the system and adding an additional constant offset term to the measurement.

Let the nominal fault-free state-space model of the system under consideration be

$$
\Sigma : \begin{cases}
  z_{k+1} = Az_k + Bu_k + T\xi_k \\
y_k = Cz_k + Du_k + \eta_k
\end{cases}
$$

with $z_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the input to the system, $y_k \in \mathbb{R}^p$ is the measured output of the system, and $\xi_k \in \mathbb{R}^{n_x}$ and $\eta_k \in \mathbb{R}^p$ are zero mean white processes with not necessarily known covariance matrices.
Then the parametrized state-space model to represent sensor and actuator faults is given as follows

\[
S : \begin{cases}
    x_{k+1} = Ax_k + BS_c u_k + b + T \xi_k \\
    y_k = S_o C x_k + S_o D S_c u_k + c + \eta_k.
\end{cases}
\]

(1)

where \(x_k \in \mathbb{R}^n\) is the state vector, \(b \in \mathbb{R}^n\) and \(c \in \mathbb{R}^p\) are offsets in the state and in the output equation, which together with \(S_c\) and \(S_o\)

\[
S_c = \text{diag}\{s^c_1, \ldots, s^c_m\}, \quad S_o = \text{diag}\{s^o_1, \ldots, s^o_p\},
\]

with \(0 \leq s^c_i \leq 1\) and \(0 \leq s^o_i \leq 1\), represent the sensor and actuator faults present in the system.

**Problem Formulation**

Consider the parametrization of sensor and actuator faults as described above. The problem is then formulated as follows: Given the matrices \(A, B, C,\) and \(D\), and input/output measurements \(\{y_k, u_k\}_{k=1}^{T+N-1}\) from system, that are related as in (1), with entries in the diagonal matrices \(S_c, S_o\), and the vectors \(b, c\) constant and unknown,

**Part (1):** find the conditions that enable the unique determination of the unknown quantities in (1) from a state-space model that models the deterministic part of (1) exactly up to a similarity transformation.

**Part (2):** estimate the unknown constants from an identified model of the deterministic part of (1) via subspace identification.

### 3 Fault Isolation

This section starts with addressing **Part (1)** of the problem formulated in Section 2. In particular, we will see that faults affecting at the same time all sensors and all actuators cannot be isolated. Although this is not a restrictive case at all, we will show that even this assumption can be released if we know the status of just one sensor (or actuator)\(^1\). The offset vectors \(b, c\) are not considered in this section since (as will be seen further in the paper) they depend linearly on the offsets in the identified model and can always be uniquely found. Thus, they play no role in the isolability conditions derived in this section.

The input/output behavior of the faulty system, not considering the offsets, can based on the state-space model (1) be denoted as

\[
H_f(z) = S_o (C(zI - A)^{-1} B + D) S_c.
\]

Let the transfer function of the nominal system, i.e. the system with no faults, be denoted as

\[
H(z) = C(zI - A)^{-1} B + D = \begin{bmatrix}
H_{11} & \cdots & H_{1m} \\
\vdots & \ddots & \vdots \\
H_{p1} & \cdots & H_{pm}
\end{bmatrix}
\]

Then we have

\[
H_f(z) = \begin{bmatrix}
s^c_1 s^o_1 H_{11} & \cdots & s^c_1 s^o_m H_{1m} \\
\vdots & \ddots & \vdots \\
s^c_p s^o_1 H_{p1} & \cdots & s^c_p s^o_m H_{pm}
\end{bmatrix}.
\]

It is then clear that, given input-output data, one can only identify the products \(s^c_i s^o_j\), \(i = 1, \ldots, p, j = 1, \ldots, m\), but not the individual entries \(s^c_i, s^o_j\). Looking at input-output data, and given \(H_{ij}\), one can only come up with a model of the form

\[
\hat{H}_f(z) = \begin{bmatrix}
\hat{\mu}_{11} H_{11} & \cdots & \hat{\mu}_{1m} H_{1m} \\
\vdots & \ddots & \vdots \\
\hat{\mu}_{p1} H_{p1} & \cdots & \hat{\mu}_{pm} H_{pm}
\end{bmatrix}
\]

where \(\hat{\mu}_{ij}\) are some estimates of the quantities \(\mu_{ij}(= s^c_i s^o_j)\).

Let us assume that we have found the estimates \(0 \leq \hat{\mu}_{ij} \leq 1\) (recall that \(0 \leq s^c_i, s^o_j \leq 1\)). The problem that we consider is as follows.

\(^1\)This would be the case when we use an intelligent sensor (or actuator), capable to diagnose and output its own status.
Problem 3.1. Given the quantities \( \mu_{ij} \) and the non-zero quantity \( s^i_c \), \( i \in \{1, \ldots, p\} \) (or \( s^j_o \), \( j \in \{1, \ldots, m\} \)), solve the following constraint non-linear least squares optimization problem

\[
\min_{s^i_c, s^j_o} \left\| \begin{bmatrix} s^1_c s^1_c & \cdots & s^j_c s^j_c & \cdots & s^m_c s^m_c \end{bmatrix} - \begin{bmatrix} \mu_{11} & \cdots & \mu_{ij} & \cdots & \mu_{pm} \end{bmatrix} \right\|_2^2
\]

under the constraints \( 0 \leq s^i_c \leq 1 \) and \( 0 \leq s^j_o \leq 1 \).

Unfortunately, this result does not always have a unique solution. The next result tells us that if we know the value of one element \( s^i_c \) of \( S_c \) (or one element \( s^j_o \) of \( S_o \)), the problem does have a unique solution.

Theorem 3.1. Problem (3.1) has a unique solution if the value of (at least) one non-zero element \( s^i_c \) of \( S_c \) (or one non-zero element \( s^j_o \) of \( S_o \)) is known.

Proof. See [12]. \( \square \)

Based on the discussion above we assume, without loss of generality, that \( s^i_c \) is known. The next result presents the (unique) solution to Problem (3.1).

Theorem 3.2 (Analytical Solution to Problem (3.1)). Consider the optimization problem

\[
\min_{s^i_c, s^j_o} \left\| \begin{bmatrix} s^1_c s^1_c & \cdots & s^j_c s^j_c & \cdots & s^m_c s^m_c \end{bmatrix} - \begin{bmatrix} \mu_{11} & \cdots & \mu_{ij} & \cdots & \mu_{pm} \end{bmatrix} \right\|_2^2
\]

subject to \( 0 \leq s^i_c \leq 1 \) and \( 0 \leq s^j_o \leq 1 \), and assume that \( 0 \leq \mu_{ij} \leq \mu_{ii} \) for \( \forall i = 1, \ldots, p \), \( j = 1, \ldots, m \). Then the unique solution to (3) is given by

\[
s^k_c = \sum_{i=1}^p \frac{\mu_{ik}}{\mu_{ii}}
\]

(4)

\[
s^i_o = \frac{\sum_{j=2}^m \mu_{ij} \left( \sum_{i'=1}^p \frac{\mu_{ij'}}{\mu_{ii'}} \right) + \mu_{ii}}{1 + \sum_{j=2}^m \left( \sum_{i'=1}^p \frac{\mu_{ij'}}{\mu_{ii'}} \right)^2}
\]

(5)

Proof. See [12]. \( \square \)

Remark 3.1. The assumption that \( 0 \leq \mu_{ij} \leq \mu_{ii} \) for \( \forall i = 1, \ldots, p \), \( j = 1, \ldots, m \) is just technical and assures that the (uniquely found) estimates \( \hat{s}^i_c \) and \( \hat{s}^j_o \) lie in the interval \([0, 1]\). In the practical implementation of the algorithm this assumption should not necessarily hold as the estimates \( \hat{s}^i_c \) and \( \hat{s}^j_o \) can get values slightly greater than one or slightly less than zero.

4 FAULTY STATE-SPACE MODEL IDENTIFICATION

In the previous section we showed under what conditions we have a unique solution to problem (3.1), found their unique solutions. Based on this analysis we will now treat Part (2) of the problem formulated in Section 2.

Given input/output measurements \( \{u_k, y_k\}_{k=T}^{T+Nr-1} \) of the system represented by (1), we can use the existing LTI subspace identification methods [21] to derive a state-space model (with state in a different coordinate basis) of the faulty
system. This is because in the time interval \([T, T + N_T - 1]\) the faults were assumed to be constant. Let this estimated model be denoted by

\[
\begin{align*}
  x_{k+1}^* &= A_k x_k^* + B_k u_k + b_k \\
y_k &= C_k x_k^* + D_k u_k + c_k.
\end{align*}
\] (6)

Assume perfect estimates to be available of the system matrices in the model of the deterministic part of the faulty system (1), and let \(T \in \mathbb{R}^{n \times n}\) be a non-singular matrix. Then the following relationships hold

\[
\begin{align*}
  TA_e &= AT \\
  BS_c &= TB_c \\
  S_o C &= C e T^{-1} \\
  S_o D S_c &= D e \\
  b &= T b_e \\
  c &= c_e
\end{align*}
\] (7) - (12)

Because of numerical errors the identified model (6) is never perfect, and therefore the strict equalities above should be approximate equalities. However, we will write them as equalities, bearing in mind that the solution that we will present is a least-squares type of solution.

The purpose is now given \(A_e, B_e, C_e, D_e, b_e, c_e\) to estimate \(T, S_c, S_o, b\) and \(c\) defined in (1). If the matrix \(T\) would be known, the problem would be trivially solved.

Let us first concentrate on the problem of deriving \(S_o\) and \(S_c\) from equations (7) - (9). First, we need to make the following (mild) assumption.

**Assumption 4.1.** Both \((A, B, C, D)\) and \((A_e, B_e, C_e, D_e)\) are minimal realizations.

Define the controllability \((C)\) and \((C_e)\) and the observability \((O)\) and \((O_e)\) matrices of the nominal model and of the identified model as follows

\[
O = \begin{bmatrix} C \\ C A \\ \vdots \\ C A^{n-1} \end{bmatrix}, \quad O_e = \begin{bmatrix} C_e \\ C_e A_e \\ \vdots \\ C_e A_e^{n-1} \end{bmatrix}, \quad C = \begin{bmatrix} B \\ A B \\ \vdots \\ A^{n-1} B \end{bmatrix}, \quad C_e = \begin{bmatrix} B_e \\ A_e B_e \\ \vdots \\ A_e^{n-1} B_e \end{bmatrix}.
\] (13)

Under the assumption (4.1) we have that

\[
\text{rank}(OC) = \text{rank}(O_e C_e) = n.
\]

By the relationships (7) to (9), we have that

\[
\begin{bmatrix} S_o C \\ S_o CA \\ \vdots \\ S_o C A^{n-1} \end{bmatrix} \begin{bmatrix} BS_c \\ ABS_c \\ \vdots \\ A^{n-1} BS_c \end{bmatrix} = \begin{bmatrix} C_e \\ C_e A_e \\ \vdots \\ C_e A_e^{n-1} \end{bmatrix} \begin{bmatrix} B_e \\ A_e B_e \\ \vdots \\ A_e^{n-1} B_e \end{bmatrix}.
\] (14)

We need to solve this non-linear matrix equation with respect to \(S_c\) and \(S_o\). Denote

\[
B = \begin{bmatrix} b_1 & b_2 & \cdots & b_m \end{bmatrix}, \quad C = \begin{bmatrix} c_1^T \\ c_2^T \\ \vdots \\ c_p^T \end{bmatrix}^T
\]

\[
B_e = \begin{bmatrix} b_{e,1} & b_{e,2} & \cdots & b_{e,m} \end{bmatrix}, \quad C_e = \begin{bmatrix} c_{e,1}^T \\ c_{e,2}^T \\ \vdots \\ c_{e,p}^T \end{bmatrix}^T
\]

and then define

\[
O_i = \begin{bmatrix} c_i \\ c_i A \\ \vdots \\ c_i A^{n-1} \end{bmatrix}, \quad O_{e,i} = \begin{bmatrix} c_{e,i} \\ c_{e,i} A_e \\ \vdots \\ c_{e,i} A_e^{n-1} \end{bmatrix}, \quad C_i = \begin{bmatrix} b_i \\ A b_i \\ \vdots \\ A^{n-1} b_i \end{bmatrix}, \quad C_{e,i} = \begin{bmatrix} b_{e,i} \\ A_e b_{e,i} \\ \vdots \\ A_e^{n-1} b_{e,i} \end{bmatrix}.
\] (15)
Then we have $O_{i}C_{j}s_{i}^{j}s_{c}^{j} = O_{i}C_{j}$ for $i = 1, \ldots, p$ and $j = 1, \ldots, m$. Application of the vec operator leads to

$$\text{vec}(O_{i}C_{j}s_{i}^{j}s_{c}^{j}) = \text{vec}(O_{i}C_{j}),$$
and we thus arrive at

$$\begin{bmatrix}
\text{vec}(O_{1}C_{1}) \\
\text{vec}(O_{1}C_{2}) \\
\vdots \\
\text{vec}(O_{n}C_{m})
\end{bmatrix} = 
\begin{bmatrix}
\delta_{e,1}^{1} \\
\delta_{e,2}^{2} \\
\vdots \\
\delta_{e,m}^{m}
\end{bmatrix} = 
\begin{bmatrix}
\text{vec}(O_{e,1}C_{1,e}) \\
\text{vec}(O_{e,1}C_{2,e}) \\
\vdots \\
\text{vec}(O_{e,m}C_{m,e})
\end{bmatrix}.$$

Therefore, taking

$$\mu = \begin{bmatrix}
\mu_{11} \\
\mu_{12} \\
\vdots \\
\mu_{pm}
\end{bmatrix} = \begin{bmatrix}
\text{vec}(O_{1}C_{1}) \\
\text{vec}(O_{1}C_{2}) \\
\vdots \\
\text{vec}(O_{p}C_{m})
\end{bmatrix} \Rightarrow \begin{bmatrix}
\text{vec}(O_{e,1}C_{1,e}) \\
\text{vec}(O_{e,1}C_{2,e}) \\
\vdots \\
\text{vec}(O_{e,m}C_{m,e})
\end{bmatrix},$$

we result in problem (3.1), the solution to which was already analyzed in Section 3.

It just remains to find estimates of $b$, $c$. By using the Kroneker product, the relationships

$$\text{diag}[S_{c}] \begin{bmatrix}
C \\
CA \\
\vdots \\
CA^{p-1}
\end{bmatrix} T = \begin{bmatrix}
C_{e} \\
C_{e}A_{e} \\
\vdots \\
C_{e}A_{e}^{p-1}
\end{bmatrix},$$

denoted as $O_{F}T = O_{e}$, and,

$$\begin{bmatrix}
B \\
AB \\
\vdots \\
A^{p-1}B
\end{bmatrix} \text{diag}[S_{c}] = T \begin{bmatrix}
B_{e} \\
A_{e}B_{e} \\
\vdots \\
A_{e}^{p-1}B_{e}
\end{bmatrix},$$

denoted as $C_{F} = TC_{e}$, can be rewritten as,

$$\begin{bmatrix}
I \\
\otimes O_{F} \\
C_{e}^{T} \otimes I
\end{bmatrix} \text{vec}(T) = \begin{bmatrix}
\text{vec}(O_{e}) \\
\text{vec}(C_{e})
\end{bmatrix}.$$

The underbraced matrix in this set of equations has full column rank and therefore a unique solution vec$(T)$ exists. Based on this solution we use (11) and (12) to find $b$ and $c$.

5 SIMULATION EXAMPLE

In this simulation example we consider a linearized model of the lateral directional dynamics of an F-14 aircraft [1]. The aircraft has two control inputs, namely the differential stabilizer deflection $\delta_{ds}$ and the rudder deflection $\delta_{dr}$. It has three measured outputs: the roll rate $p$, the yaw rate $r$, the lateral acceleration $y_{ac}$, and a side-slip angle (which is in fact a calculated variable rather than a measured one) $\beta$. The aircraft model has four states: lateral velocity $\nu$, yaw rate $r$, roll rate $p$, and roll angle $\phi$. It is represented in state-space by the following nominal (fault-free) model

$$\begin{bmatrix}
x_{k+1} \\
y_{k}
\end{bmatrix} = 
\begin{bmatrix}
A & B & \xi_{k} \\
C & D & \eta_{k}
\end{bmatrix} \begin{bmatrix}
x_{k} \\
y_{k}
\end{bmatrix},$$

with $x = [\nu \ r \ \dot{p} \ \dot{\phi}]^{T}$, $u = [\delta_{ds} \ \delta_{dr}]^{T}$, and $y = [\beta \ p \ r \ y_{ac}]^{T}$. For the numerical values of the matrices $A$, $B$, $C$, and $D$ the reader is referred to [1].

The state-space representation of our faulty system is

$$\begin{bmatrix}
x_{k+1} \\
y_{k}
\end{bmatrix} = 
\begin{bmatrix}
A_{x} & B_{x} & \xi_{k} \\
S_{p}C & A_{x} & \eta_{k}
\end{bmatrix} \begin{bmatrix}
x_{k} \\
y_{k}
\end{bmatrix},$$

with simultaneous sensor and actuator faults represented by

$$S_{c} = \text{diag}\{0.5, 0\}, \ S_{o} = \text{diag}\{0.5, 0.7, 0.6, 0.3\}.$$
All of the system outputs are corrupted measurement noise resulting in SNR of 16 dB. The inputs were corrupted by noise, so that their corresponding SNR’s were 25 dB.

Note, that we thus simulate faults in all of the sensors and actuators at the same time. Therefore, recalling theorem (3.1), this problem is solvable if we know (at least) one non-zero diagonal element of one of the matrices $S_e$ and $S_o$. We then assume that we know that $S_o^1 = 0.5$ (Note that $S_e^1$ weights a computed output rather than a measured one).

We next collect input/output data (with data length $N_F = 1000$) from system (17), corrupted by measurement noise of 16 dB for each output, and process noise of 40 dB for each input. This data we process using the SMI toolbox for MATLAB [22], and we obtained an identified model of the form in equation (6). We then follow the algorithm presented in Section 4 to obtain the following estimates for the faults

\[
\hat{S}_e = \text{diag}\{0.5011, 0.0004\}, \hat{S}_o = \text{diag}\{0.4885, 0.7, 0.6013, 0.2967\}. \quad (19)
\]

Clearly, we obtained very precise estimates even in the presence of the large process and measurement noises simulated in this numerical example.

In order to graphically visualize the precision of the estimates of the faults in the system, in Figure 1 we depict, for some input/output channels, the magnitude plot of system (17) parametrized by the real faults, and the same system (17) parametrized by the estimated faults. We have depicted only the channels corresponding to the first control action-the reason for that is due to the simulated complete loss of the second actuator. The plot shows perfect matching of the magnitude plots of the two systems, implying that the identified faults represent a system with practically the same input/output behaviour as the real faulty system. Note that the scaling on the $y$-axis of Figure 1 (d) is $10^{-3}$. Therefore, in addition to fault isolation this identified model could subsequently be used in the controller reconfiguration step. The key element under investigation is to use subspace identification that enables to obtain consistent estimates under closed-loop experimental conditions.

6 CONCLUSIONS

In this paper we presented a new approach for simultaneous sensor and actuator faults in linear discrete time-invariant MIMO systems, given input/output measurements. We model the faults as scaling factors for the input and the output of the system plus offset terms. Given measured data from the system we used the subspace model identification tools to identify a state-space model of the faulty system, which however is in a different state basis. From this model we extract our estimates for the sensor and actuator faults present in the system, which is an algorithm that consists of solving a non-linear least squares optimization problem for the scaling factors, and a linear least squares optimization problem for the offset terms. Unique analytical solution to the non-linear optimization problem is derived, and is shown to exist under the assumption that either there is one sensor and one actuator operating normally, or that the status of one sensor (or one actuator) is known. The approach was tested on a numerical example with a linearized model of the lateral directional dynamics of an F-14 aircraft, in which simultaneous faults in all sensors and actuators were simulated.

REFERENCES


$^2$We remind that if there was at least one healthy (not failed) sensor or actuator then this assumption drops out. However, with this example we treat the most general case of Problem (3.1).


