Abstract: A method is proposed for estimating both the weights and the state of a multiple model system with one common state vector. In this system, the weights are related to the activation of each individual model. For the resulting nonlinear estimation problem a method is developed that efficiently decomposes the total problem into two separate parts, one for estimating the model weights and one for estimating the state. The method has been validated on a component, actuator and sensor fault detection and identification problem for a linearized model of an aircraft. Copyright © 2006 IFAC

Keywords: Fault detection and identification, state-space methods, recursive estimation, filtering theory, numerical algorithms

1. INTRODUCTION

The field of Fault Detection and Identification (FDI) has received much attention from researchers in the past decades. The research in this field was driven by the increasing demand of reliability in control systems. Numerous research efforts resulted in a vast number of different methods for FDI. A good overview and discussion of these methods can be found in (Patton et al., 2000) and (Blanke et al., 2003).

One of the many different approaches to FDI, is the multiple model approach. A multiple model system consists of a model set that contains local models each corresponding to a specific condition of the system. In an FDI setting, the local models usually represent different fault conditions of the monitored system. Furthermore, a model set to be used for FDI also contains the fault-free model of the system. When the system is in its normal operation mode (no faults), the model corresponding to the fault-free condition will have maximum activation, which corresponds to a weight of one, and all other models in the model set will have a weight of zero (minimum activation). In case of a fault, the corresponding local model will have a weight greater than zero. In general, one or more of the local models in a model set can have weights greater than zero. Some general directions for model set design are given in (Li et al., 2005).

Different research efforts have been made in the field of FDI with multiple models, see e.g. (Zhang and Li, 1998), (Hanlon and Maybeck, 2000) and the references therein. The motivation for using multiple model systems for FDI stems from the fact that a large class of fault conditions can be modeled, contrary to many other FDI methods that can only be applied to a limited number of fault conditions. Most classical multiple model FDI methods in the literature use a bank of filters in which each filter is based on one of the models from the model set. The weights of these models are computed by using the residuals of each filter and their assumed probability distributions in combination with Bayesian theory.

The multiple model methods using banks of filters are especially suitable for determining which of the local models best matches the system. The weight of the model “closest” to the true system, converges to one. Therefore, for FDI purposes, partial faults are
estimated by using approaches such as discretization (Fisher and Maybeck, 2002) or hierarchical structures in combination with discretization (Ru and Li, 2003). The drawback of these approaches is that extra models are required to represent the partial faults, and despite the extra models, the fault identification may still suffer from quantization errors (Fisher and Maybeck, 2002). Furthermore, the methods based on the Interacting Multiple Model (IMM) algorithm (Zhang and Li, 1998) have the disadvantage that a matrix with transition probabilities must be chosen. Choosing this matrix, which is required for re-initialization of the different filters, can be very involved in practice (Fisher and Maybeck, 2002), (Kanev, 2004).

In this paper an alternative approach is used in which all local models have one common state vector. In this framework, the system matrices are represented by a convex combination of the state-space matrices of the local models. Contrary to classical methods using banks of filters, only one filter is required to estimate both the state and the weights of the local models. This filter, however, is nonlinear. A method is proposed that efficiently solves this nonlinear filtering problem by splitting it into two subproblems: one nonlinear optimization problem with only the weight vector as variable and one linear problem for state estimation.

Although the algorithm proposed in this paper is motivated by FDI, this is not the only field of application. This algorithm can be used in the same manner as the classical multiple model algorithms for other applications. Examples of such applications are, but not limited to, multiple model adaptive control (Athans et al., 2005), fault-tolerant control (Zhang and Jiang, 2001) and approximation of nonlinear systems with multiple models (Banerjee et al., 1997).

This paper is organized as follows. Section 2 formulates the problem of combined weight and state estimation. In Section 3 the algorithm developed for solving this problem is explained. Section 4 presents the simulation results. Finally, Section 5 provides the conclusions and suggestions for further research.

2. PROBLEM FORMULATION

This paper considers a multiple model system with one common state vector. With this multiple model system, sensor faults, actuator faults and component faults can be modeled. The system is given by

\[ x_{k+1} = \sum_{i=1}^{N} \mu_k^{(i)} [A^{(i)} x_k + B^{(i)} u_k] + w_k \]  
\[ y_k = \sum_{i=1}^{N} \mu_k^{(i)} [C^{(i)} x_k + D^{(i)} u_k] + v_k \]  
\[ \mu_k^{(i)} \geq 0, \quad \sum_{i=1}^{N} \mu_k^{(i)} = 1, \]  

where \( \mu_k^{(i)} \in [0, 1] \) is the model weight corresponding to the \( i \)th model represented by the state space quadruple \( \{A^{(i)}, B^{(i)}, C^{(i)}, D^{(i)}\} \), \( x_k \in \mathbb{R}^n \) is the state, \( u_k \in \mathbb{R}^m \) is the input, \( y_k \in \mathbb{R}^l \) is the output, \( w_k \in \mathbb{R}^n \) is the process noise, \( v_k \in \mathbb{R}^l \) is the measurement noise and \( N \) is the number of local models. Both \( v_k \) and \( w_k \) are assumed to be zero-mean white noise sequences.

In order to obtain a more general representation of these sequences, the general covariance representation is introduced. In this representation the process and measurement noise are denoted as

\[ \begin{pmatrix} v_k \\ w_k \end{pmatrix} = \begin{pmatrix} R_k & 0 \\ 0 & Q_k \end{pmatrix} \begin{pmatrix} \tilde{v}_k \\ \tilde{w}_k \end{pmatrix} \]  

where \( R_k \) is the square root of the measurement covariance matrix and \( Q_k \) is the square root of the process covariance matrix. \( \tilde{v}_k \) and \( \tilde{w}_k \) are uncorrelated zero mean white noise sequences with unit variance.

A schematic representation of the evolution of the state of the described multiple model system is given in Fig. 1. Here, it can be seen that the common state \( x_{k+1} \) is a weighted sum of the contributions of each of the local state vectors \( (x_k^{(1)}, \ldots, x_k^{(N)}) \). The weights corresponding to the different models are included in the vector \( \mu_k = [\mu_k^{(1)} \mu_k^{(2)} \cdots \mu_k^{(N)}]^T \). The difference with multiple model systems used in classical filter banks, is that only the common state vector is used to compute the state at the next time instant, and not the local state vectors.

The problem of interest in this paper is the estimation of both the state \( x_{k+1} \) and the weight vector \( \mu_k \) of the system described by (1)-(3) given a model set and a set of input/output data. This estimation problem is nonlinear due to the product of the state and the weight vector. An efficient algorithm for this nonlinear problem is developed in the following section.

3. ALGORITHM FOR STATE AND MODEL WEIGHT ESTIMATION

In this section, the algorithm for state and weight estimation is formulated using square root covariance filtering theory (Verhaegen and van Dooren, 1986) because it allows easy manipulation of the different filtering steps, as will become clear in the following.
3.1 Standard algorithm

Using the notation from (4), the filtering problem for the multiple model system at time instant $k$ can be compactly written in the following form

$$\begin{bmatrix} \hat{x}_{k|k−1} \\ y_k - D_{\mu_k} u_k \\ -B_{\mu_k} u_k \end{bmatrix} = \begin{bmatrix} A_{\mu_k} & 0 & 0 \\ C_{\mu_k} & 0 & 0 \\ I_n & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k+1} \end{bmatrix} + \begin{bmatrix} S_{k|k−1} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ v_k \end{bmatrix} + \begin{bmatrix} R_k \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{v}_k \\ \tilde{w}_k \end{bmatrix} \quad (5)$$

where in the first block row the generalized covariance representation of the state estimate is included in the form $\hat{x}_{k|k−1} = \hat{x}_k + S_{k|k−1}\tilde{x}_k$. In this representation, $\hat{x}_{k|k−1}$ is the estimated state at time instant $k$ given the data up to time instant $k-1$. $S_{k|k−1}$ is the estimation error covariance matrix and $\hat{x}_k$ is the corresponding unit variance noise sequence. The matrix $A_{\mu_k}$ is a shorthand notation for $\sum_{i=1}^{N} \mu_k^{(i)} A^{(i)}$. The system matrices $B_{\mu_k}, C_{\mu_k}$ and $D_{\mu_k}$ are defined similarly. Let (5) be given in a shorthand notation by

$$Y_{\mu_k} = F_{\mu_k} X_k + L_k U_k$$

the solution to the estimation problem of the state and model weights is found by recursively solving the following optimization problem

$$\min_{\mu_k, X_k} U_k^T U_k$$

s.t. $Y_{\mu_k} = F_{\mu_k} X_k + L_k U_k$, $\mu_k^{(i)} \geq 0 \forall i, \sum_{i=1}^{N} \mu_k^{(i)} = 1$ \quad (7)

In order to remove the cross-products between $\mu_k$ and the state vector $X_k$, the following invertible left transformation is applied to (5)

$$T_{\mu_k} = \begin{bmatrix} C_{\mu_k} - I_n & 0 \\ I_n & 0 \end{bmatrix}$$

It should be noted that applying an invertible left transformation does not change the problem because the residual $U_k^T U_k$ remains unchanged. After left multiplication with $T_{\mu_k}^{-1}$, (5) becomes

$$\begin{bmatrix} C_{\mu_k} \hat{x}_{k|k−1} + D_{\mu_k} u_k - y_k \\ A_{\mu_k} \hat{x}_{k|k−1} + B_{\mu_k} u_k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ I_n & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k+1} \end{bmatrix} + \begin{bmatrix} C_{\mu_k} S_{k|k−1} & -R_k \\ A_{\mu_k} S_{k|k−1} & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ \tilde{v}_k \\ \tilde{w}_k \end{bmatrix} \quad (9)$$

in which it can be observed that the first row of $T_{\mu_k} F_{\mu_k}$ contains zeros. This fact can be exploited by using the following theorem.

**Theorem 1: Reduction of the number of optimization variables**

The optimization problem

$$\mu_k, X_k^* = \arg \min_{\mu_k, X_k} \mathcal{U}_k^T \mathcal{U}_k$$

s.t. $\begin{bmatrix} Y_{\mu_k}^1 \\ Y_{\mu_k}^2 \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix} X_k + \begin{bmatrix} L_{\mu_k}^1 & 0 \\ L_{\mu_k}^2 & T_{\mu_k} \end{bmatrix} \begin{bmatrix} \mathcal{U}_k^1 \\ \mathcal{U}_k^2 \end{bmatrix}$ \quad (10)

$$\mu_k^{(i)} \geq 0 \forall i, \sum_{i=1}^{N} \mu_k^{(i)} = 1$$

where $\mathcal{U}_k^T = [(\mathcal{U}_k^1)^T (\mathcal{U}_k^2)^T]$, is equivalent to

$$\mu_k^* = \arg \min_{\mu_k} \| (L_{\mu_k}^1)^{-1} Y_{\mu_k}^1 \|^2_2$$

s.t. $\mu_k^{(i)} \geq 0 \forall i, \sum_{i=1}^{N} \mu_k^{(i)} = 1$ \quad (11)

$$X_k^* = Y_{\mu_k^*}^2 - L_{\mu_k^*}^2 (L_{\mu_k^*}^1)^{-1} Y_{\mu_k^*}^1$$

**Proof**: The residual $\mathcal{U}_k^T \mathcal{U}_k$ can be written as

$$\left\| (L_{\mu_k}^1)^{-1} Y_{\mu_k}^1 \right\|^2_2 + \left\| (L_{\mu_k}^2)^{-1} (Y_{\mu_k}^2 - X_k - L_{\mu_k}^2 (L_{\mu_k}^1)^{-1} Y_{\mu_k}^1) \right\|^2_2$$

in which the relation $\mathcal{U}_k^2 = (L_{\mu_k}^1)^{-1} Y_{\mu_k}^1$ has been used in the second block row. The matrices $L_{\mu_k}^1$ and $L_{\mu_k}^2$ should be invertible matrices. Subsequently, (13) can be equivalently split up in the sum of two residuals in the following way

$$\mathcal{U}_k^T \mathcal{U}_k = \left\| (L_{\mu_k}^1)^{-1} Y_{\mu_k}^1 \right\|^2_2 + \left\| (L_{\mu_k}^2)^{-1} (Y_{\mu_k}^2 - X_k - L_{\mu_k}^2 (L_{\mu_k}^1)^{-1} Y_{\mu_k}^1) \right\|^2_2$$

In (14) it can be seen that the first part of the residual is only dependent on $\mu_k$ and that the second part is dependent on both $\mu_k$ and $X_k$. Minimizing the total residual can therefore be done by first minimizing the first part with only $\mu_k$ as an optimization variable. Then the resulting value of $\mu_k$ can be used to compute a value for $X_k$ that makes the second part of the residual zero. The minimization of the first part of the residual is done in (11) and the value of $X_k$ that makes the second part zero is computed linearly with (12).

Let $T_{\mu_k} L_k$ be an orthogonal right transformation matrix that makes $T_{\mu_k} L_k$ the covariance matrix in (9) lower triangular. Note that because $T_{\mu_k} L_k$ is orthogonal, the size of the residual does not change. After right multiplication of the covariance matrix in (9) with $T_{\mu_k}^{-1}$, this estimation problem is exactly in the form of the first constraint in (10). According to Theorem 1, estimating the model weights can then be done by only considering the first block row of (9) after right multiplication of the covariance matrix with $T_{\mu_k}^{-1}$. This first block row only depends on the state space matrices $C_{\mu_k}$ and $D_{\mu_k}$ and does not consider $A_{\mu_k}$ and $B_{\mu_k}$. Hence, when the measurement equation is not dependent on $\mu_k$ (i.e. $C_{\mu_k}$ and $D_{\mu_k}$ are constant) and the state equation is, the estimator will not be able to correctly reproduce the state and weights. A method to prevent this undesirable property is presented in the next part.
A modification of the standard filtering algorithm is performed by assuming that $\mu_k = \mu_{k-1}$ for the state equation. This assumption does not restrict variation of $\mu_k$, because $\mu_k$ is computed at each time instant with different data. A result of this assumption is that it allows the state equation at time instant $k-1$ and its generalized covariance expression to be formulated as follows

$$
x_k = A_{\mu_k} x_{k-1} + B_{\mu_k} u_{k-1} + Q_{k-1} \tilde{w}_{k-1}
$$

(substituting $x_k$ in (15) by (16) results in)

$$
x_k = A_{\mu_k} \hat{x}_{k-1} - 1 - 2 + B_{\mu_k} u_{k-1} + Q_{k-1} \tilde{w}_{k-1}
$$

(17)

The two noise terms in this expression can be written in a shorthand notation as

$$
Q_{k-1} \tilde{w}_{k-1} - A_{\mu_k} S_{k-1|k-2} = Q_{k-1} \tilde{w}_{k-1}
$$

where $Q_{k-1}$ can be shown to be equal to $[Q_{k-1} - A_{\mu_k} S_{k-1|k-2}] [Q_{k-1} - A_{\mu_k} S_{k-1|k-2}]^T$. Combining (17) and (18) with the original estimation problem from (5) results in

$$
[-A_{\mu_k} \hat{x}_{k-1} - 1 - 2 - B_{\mu_k} u_{k-1} y_k - D_{\mu_k} u_{k-1} - B_{\mu_k} u_{k-1}]
$$

$$
[Q_{k-1} 0 0 0 0 0 S_{k-1|k-1} 0 0 0 0 R_0 0 0 0 Q_k]
$$

This expression can be put in the form required for Theorem 1, by first applying the invertible left transformation

$$
T_{r,2} = \begin{bmatrix}
I_n & I_n & 0 & 0 \\
0 & C_{\mu_k} & -I_n & 0 \\
0 & I_n & 0 & 0 \\
0 & A_{\mu_k} & 0 & -I_n
\end{bmatrix}
$$

(20)

which results in the following expression

$$
\begin{bmatrix}
\hat{x}_{k|k-1} - A_{\mu_k} \hat{x}_{k|k-2} - B_{\mu_k} u_{k-1} \\
C_{\mu_k} \hat{x}_{k|k-1} + D_{\mu_k} u_{k-1} - y_k \\
A_{\mu_k} \hat{x}_{k|k-1} + B_{\mu_k} u_{k-1}
\end{bmatrix}
$$

$$
\begin{bmatrix}
Q_{k-1} 0 0 0 0 0 S_{k-1|k-1} 0 0 0 0 R_0 0 0 0 Q_k
\end{bmatrix}
$$

(21)

Subsequently, an orthogonal right transformation $T_{r,2}$ is applied to the covariance matrix in (21) that makes it lower triangular. The form required for Theorem 1 is then obtained. According to this theorem, the estimation of $\mu_k$ can be performed using the first two block rows of (21) (after application of $T_{r,2}$). These two block rows do not contain dependencies on all system matrices. Therefore, contrary to the standard algorithm, weights of local models that only differ in any one of the four system matrices can be estimated with the modified algorithm.

**Remark 1:** The developed algorithm is recursive. Therefore, at time instant $k$ a value for $S_{k+1|k}$, to be used at time instant $k+1$, should be computed. This value can be extracted from $L_{\mu_k}^2$ in (10). In order to see this, consider the bottom block row of the first constraint in (10), which is written as

$$
X_k = \hat{X}_{k|k} - L_{\mu_k}^2 \hat{U}_k
$$

(22)

Here, it can be observed that $L_{\mu_k}^2$ is indeed the desired covariance matrix because (22) is exactly a generalized covariance expression of the state vector $X_k$ with a mean of $X_{k|k}$ (which is equal to (12)) and a covariance equal to $-L_{\mu_k}^2$. Since $X_{k+1|k}$ is in the bottom block row of $X_k$, $S_{k+1|k}$ can be found in the bottom right block of $L_{\mu_k}^2$.

**Remark 2:** An advantage of the proposed algorithm is that no pre-knowledge is required on $\mu_k$ other than the assumption that $\mu_k = \mu_{k-1}$ for the state equation. This is in contrast to, e.g. the IMM method in which a transition probability matrix is required.

The algorithm can be summarized as follows.

**Required:** $Q_k$, $R_k$ and initial conditions: $S_{1|0}$, $\hat{x}_{1|0}$. For $k = 1, 2, \ldots, n_s$ ($n_s$ is the number of available data samples)

1. Compute $\mu_k$ by numerically solving the problem of (11) using (21) and $T_{r,2}$. For the optimization, the Levenberg-Marquardt algorithm can be used.
2. $T_{r,2}$ is computed with a QR-decomposition of the covariance matrix in (21).
3. Compute $\hat{x}_{k+1|k}$ from (12) using (21) and $T_{r,2}$ and extract $S_{k+1|k}$ from $L_{\mu_k}^2$.

## 4. SIMULATION RESULTS

In this section an illustrative application of the proposed method is given. The application is a linearized dynamical model of a vertical take-off and landing (VTOL) aircraft, which was first introduced in (Narendra and Tripathi, 1973) and since then was used by various researchers as a benchmark for IMM filters, e.g. in (Zhang and Jiang, 2001).

### 4.1 Model description

For simulation purposes, the VTOL model has been discretized with a sampling time of $T = 0.02$ s, which results in the following system matrices

$$
A = \begin{bmatrix}
0.9993 & 0.0005 & 0.0003 & -0.0091 \\
0.0010 & 0.9800 & -0.0007 & -0.0796 \\
0.0020 & 0.0072 & 0.9862 & 0.0279 \\
0 & 0.0001 & 0.0199 & 1.0003
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0.0088 & 0.0035 \\
0.0702 & -0.1503 \\
-0.1094 & 0.0886 \\
-0.0011 & 0.0009
\end{bmatrix},
$$

$$
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$
The state vector contains the horizontal velocity, the vertical velocity, the pitch rate and the pitch angle. The inputs of the system are the collective pitch control, which is used for controlling the vertical motion and the longitudinal cyclic pitch control, which is used for controlling the horizontal velocity of the aircraft.

4.2 Simulation

For the simulation experiment a model set is used that consists of four models. The first model $M^{(1)}$ corresponds to the fault-free system. The second model $M^{(2)}$ models the system with a component fault, the third model $M^{(3)}$ corresponds to the system having a total fault in the fourth sensor and the fourth model $M^{(4)}$ corresponds to an actuator fault. Both $M^{(2)}$ and $M^{(4)}$ are proposed in (Zhang and Jiang, 2001) as possible fault models. The fault models are created by using the following modified versions of the corresponding matrices

$$A^{(2)} = \begin{bmatrix} 0.9993 & 0.0005 & 0.0003 & -0.0091 \\ 0.0010 & 0 & -0.0007 & -0.0796 \\ 0.0020 & 0.0072 & 0.9862 & 0.0279 \\ 0 & 0.0001 & 0.0199 & 1.0003 \end{bmatrix}$$

$$C^{(3)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \quad B^{(4)} = \begin{bmatrix} 0.0008 & 0.0035 \\ -0.0008 & -0.0748 \\ -0.0268 & 0.0225 \\ -0.0043 & 0.0034 \end{bmatrix}$$

where the changes in $A^{(2)}, C^{(3)}$ and $B^{(4)}$ with respect to the fault-free system matrices are underlined. The other system matrices are left unchanged for the three fault models.

The input/output data to be used by the algorithm proposed in Section 3 is obtained by simulating the described model set. Because the models in the model set are unstable, the system is stabilized with a state feedback controller with $u=r-K_{FB}x$, where $r$ is a reference input vector and $K_{FB}$ is a feedback gain, which has been chosen such that it ensures that $A^{(i)}-B^{(i)}K_{FB}$ is a stable system. For excitation of the system, the reference input $r$ is chosen to be a sinusoidal signal. Furthermore, the system is simulated with process and measurement noise with covariance $10^{-5}I$, the initial state is chosen as $x_{0} = [0 \ 0 \ 0 \ 0]^T$. Note that both process and measurement covariance are time invariant. The simulated faults are described in Table 1.

<table>
<thead>
<tr>
<th>Fault description</th>
<th>Time interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component fault</td>
<td>2-3.5 s</td>
</tr>
<tr>
<td>60% fault in sensor 4</td>
<td>5.5-6 s</td>
</tr>
<tr>
<td>30% fault in sensor 4</td>
<td>7-7.5 s</td>
</tr>
<tr>
<td>Actuator fault</td>
<td>9-10 s</td>
</tr>
</tbody>
</table>

The input/output data obtained by simulating the fault scenario described in Table 1 is used to evaluate the proposed filter. The filter is initialized with state $\hat{x}_{1|0} = [10 \ 20 \ 5 \ 5]^T$, the covariance matrices are chosen as $Q_k = 10^{-1}I$ and $R_k = 10^{-3}I$. The initial value for the estimation error covariance matrix is chosen as $S_{1|0} = Q_k$. Note that all these quantities are different from the ones with which the system is simulated. This is done in order to evaluate the robustness of the proposed method with respect to wrong initial conditions and wrong choice of the covariance matrices. The results of the model weight estimation are depicted in Fig. 2. In this figure, the estimated weights corresponding to the four models from the model set are depicted. The darker areas in this figure, indicate the intervals in which a deviation from the fault-free condition can be expected. The weights $\mu^{(1)}, \mu^{(2)}, \mu^{(3)}$ and $\mu^{(4)}$ correspond to the weights of models $M^{(1)}, M^{(2)}, M^{(3)}$ and $M^{(4)}$, respectively. It can be seen in Fig. 2 that in the time interval 2-3.5 s, $\mu^{(1)}$ becomes 0 and $\mu^{(2)}$ becomes 1. This corresponds exactly to the component fault that is simulated in this interval. In the time interval 5.5-6 s, $\mu^{(1)}$ becomes 0.4 and $\mu^{(3)}$ becomes 0.6. This corresponds exactly to the 60% fault in the fourth sensor because $M^{(3)}$, which models a full fault in the fourth sensor, is not fully valid, but it is valid for 60%. The fault-free model still remains valid for 40% in this case. The second partial sensor fault is also correctly identified as can be derived from the observation that $\mu^{(3)}$ becomes 0.3 in the interval 7-7.5 s. At $T = 9$ s the actuator fault is inserted. This fault is correctly identified by $\mu^{(4)}$, which becomes 1 at this time instant. It can be observed that despite the wrong initial conditions and covariance matrices, the model weight estimation is generally performed well, even for the partial sensor faults for which an interpolation between two local models is required.

Because the estimated states for the above experiment almost perfectly follow the simulated states, a figure of this result has been omitted for brevity.

4.3 Discussion

In the previous simulation a component fault, partial sensor faults and an actuator fault have been identified successfully. The correct identification of the actuator
and component fault shows that the developed algorithm is capable of dealing with situations in which the measurement equations of different models in a model set are equal. Because partial occurrence of the presented component and actuator fault can not be modeled by a convex combination of the models in the model set, this type of faults is not simulated. For partial sensor faults, however, this can be done. The correct identification of the partial sensor faults shows that the algorithm does not require the use of quantized models for identifying them. A full sensor fault model included in the model set is sufficient for estimating partial faults of the corresponding sensor that may have different sizes. This is in contrast to the conventional multiple model filters such as the IMM filter (Ru and Li, 2003). For the same purpose of partial sensor fault estimation, the models of the partial faults are also included in the IMM model set. Therefore, it can be concluded that the developed algorithm has better interpolation properties than the IMM approach when used for partial sensor fault estimation.

An advantage of the IMM approach, however, is its computational speed. The reason for this is that only basic computations are required in the IMM approach. In the approach developed in this paper, an optimization procedure is run, which makes it slower than the IMM approach. Nevertheless, it can still be easily applied in many realtime purposes.

An issue that plays a role in both the proposed method and the IMM method, is the issue of model distance. If the models in the model set are not “distant” enough from each other, the proposed method can display unsatisfactory performance. For IMM this is also the case (Kanev, 2004). In the described experiment this issue did not play a role, but for model sets with a large number of models this issue can indeed rise.

5. CONCLUSIONS

A method has been proposed to estimate both the state and the weights in a multiple model system with one common state vector. The resulting nonlinear estimation problem is solved in an efficient way by separating it in a part that estimates the weights and a part that estimates the state. An advantage of the proposed method is that it has better interpolation properties than existing multiple model filters. Furthermore, no assumption on the evolution of the weights is required. The proposed method is successfully applied for the identification of a component fault, an actuator fault and partial sensor faults in a linearized model of a VTOL aircraft. Future research will focus on extending the proposed method to FDI for nonlinear systems by using linear parameter varying models (Hallouzi et al., 2005). Furthermore, the issue of “distance” between the models in the multiple model set will be investigated.

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